

# Grover search with pairs of trapped ions

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## Abstract

The desired interference required for quantum computing may be modified by the wave function oscillations for the implementation of quantum algorithms[Phys.Rev.Lett.84(2000)1615]. To diminish such detrimental effect, we propose a scheme with trapped ion-pairs being qubits and apply the scheme to the Grover search. It can be found that our scheme can not only carry out a full Grover search, but also meet the requirement for the scalable hot-ion quantum computing. Moreover, the ion-pair qubits in our scheme are more robust against the decoherence and the dissipation caused by the environment than single-particle qubits proposed before.

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## I. INTRODUCTION

Since Shor's discovery<sup>[1]</sup> of the quantum algorithm for factoring large integers, much progress has been made in the field of the quantum computing. It has been generally considered that the quantum computer distinguishes the classical computer in the capabilities to operate quantum mechanically on superpositions of quantum states and to exploit resulting interference effects. With these capabilities, quantum computers can outperform classical ones in solving classically intractable problem<sup>[1,2]</sup> or solving tractable problems more rapidly<sup>[3]</sup>. It has been proposed that several physical systems can be used for the quantum computing, such as the ion trap, nuclear magnetic resonance(NMR) system, atom-cavity interaction and so on. However, up to now, most experimental demonstrations concerning the quantum algorithm and quantum communication have been solely performed with NMR technique<sup>[4,5]</sup> due to its technique maturity. Meanwhile there are intensive dispute<sup>[6]</sup> for whether the genuine quantum computing was made in the NMR system since the manipulation with NMR technique is applied on the bulk molecules instead of the individual molecule. In contrast, in the ion trap quantum computing, the manipulation is indeed performed on individual trapped ion by quantum level, and the coupling between the electronic states of the ion and its vibrational motion is made by the laser fields. Since the success of first experiment<sup>[7]</sup> of two-qubit controlled-NOT with a single ultracold  $Be^+$  based on the proposal by Cirac and Zoller<sup>[8]</sup>, many related theoretical schemes<sup>[9]</sup> have been put forward, and the experimental progress<sup>[10,11]</sup> in this respect has also been made. However, it is hard to achieve the entanglement of large numbers of trapped ions because the experiment relied on the particular behavior of the ions<sup>[10]</sup>. Recently, an approach with bichromatic field<sup>[12]</sup> was proposed, which leads to the success of entanglement of four trapped ions<sup>[13]</sup>. In that proposal, two identical two-level ions in the string are both illuminated with two lasers of different frequencies  $\omega_{1,2} = \omega_{eg} \pm \delta$ , where  $\omega_{eg}$  is the resonant transition frequency of the ions, and  $\delta$  the detuning, not far from the trap frequency  $\nu$ . With the choice of laser detunings the only energy conserving transitions are from  $|ggn\rangle$  to  $|een\rangle$  or from  $|gen\rangle$  to  $|egn\rangle$ ,

where the first(second) letter denotes the internal state  $e$  or  $g$  of the  $i^{th}$ ( $j^{th}$ ) ion and  $n$  is the quantum state for the vibrational state of the ion. That is to say, the states  $|ggn\rangle$  and  $|een\rangle$ , separated by  $\omega_1 + \omega_2$  are resonantly coupled and so are the degenerated states  $|egn\rangle$  and  $|gen\rangle$ . As we consider  $\nu - \delta \gg \eta\Omega$  with  $\eta$  being the Lamb-Dicke parameter and  $\Omega$  the Rabi frequency, there is only negligible population being transferred to the intermediate states with vibrational quantum number  $n \pm 1$ . It has been proven that this two-photon process is nothing to do with the vibrational state  $|n\rangle$ . So the quantum computing with such configuration is valid even for the hot ions.

As we know, the implementation of the quantum computing is based on two basic operations<sup>[14]</sup>. One is the single-qubit rotation, and the other the two-qubit operation. The suitable composition of such two operations will in principle carry out any quantum computing operation we wanted. However, the quantum computing is implemented on the superposition of eigenstates of the Hamiltonian. According to the Schrödinger equation, during the time interval  $t$ , each quantum state  $\Psi_i$  acquires a phase  $-E_i t$ , where  $E_i$  is the eigenenergy of the state  $\Psi_i$ (supposing  $\hbar = 1$ ). Thus any delay time between the operations will produce unwanted different phases in different quantum states<sup>[15,16]</sup>, which modifies the quantum interference of an ideal quantum computing, and spoils the correct results we desired. How to avoid this detrimental effect? The authors of Ref.[15] proposed an ideal solid-state qubits model making use of controllable low-capacitance Josephson junction to avoid this undesired phase evolution, in which energy splitting between logic states can be tuned to be zero during the delay time. Ref.[16] carried out a general consideration on this problem and suggested to use stably continuous reference oscillations with the resonant frequency for each quantum transition in the process of the quantum computing. We consider that the most efficient approach is to use the degenerated states to be the logic states, which can transfer the relative phases to a global one and the errors caused by the relative phases would be eliminated completely.

In this contribution, we will demonstrate a scheme to pair the trapped ions to be a qubit for eliminating the detrimental effect referred to above. Our proposal is based on the

hot-ion quantum computing model of Ref.[12], by choosing the transition paths from  $|egn\rangle$  to  $|gen\rangle$ , and setting  $|eg\rangle = |0\rangle$  and  $|ge\rangle = |1\rangle$ . As the qubits  $|0\rangle$  and  $|1\rangle$  are degenerated in energy, no unwanted relative phases will appear in the delay time between any two of the operations. We will first carry out a two-qubit Grover search with our scheme, and then extend the scheme to the more-qubit cases. Finally, a discussion will be made for the implementation of a full Grover search as well as the advantage and limitation of our work.

## II. TWO QUBITS GROVER SEARCH WITH TRAPPED ION-PAIRS

In the Lamb-Dicke limit( $\eta \ll 1$ ) and weak excitation regime( $\Omega < \nu$ ), we may obtain the time evolution of the states from the second order perturbation theory with the definition of effective Rabi frequency  $\tilde{\Omega} = -\frac{(\Omega\eta)^2}{2(\nu-\delta)}$ <sup>[12]</sup>,

$$\begin{aligned}\hat{U}|1\rangle &= \cos\left(\frac{\tilde{\Omega}T}{2}\right)|1\rangle - i \sin\left(\frac{\tilde{\Omega}T}{2}\right)|0\rangle, \\ \hat{U}|0\rangle &= \cos\left(\frac{\tilde{\Omega}T}{2}\right)|0\rangle - i \sin\left(\frac{\tilde{\Omega}T}{2}\right)|1\rangle.\end{aligned}\quad (1)$$

Setting  $|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$  and  $|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ , we obtain  $\hat{U} = \hat{U}(\theta) = \begin{pmatrix} \cos\theta & -i \sin\theta \\ -i \sin\theta & \cos\theta \end{pmatrix}$  with  $\theta = \frac{\tilde{\Omega}T}{2}$ . To construct a quantum computing model, what we need to do in the following is to find a suitable two-qubit operation, like controlled-NOT gate, i.e., if and only if the first ion and the second ion are respectively in states  $|g\rangle$  and  $|e\rangle$ , the third and fourth ions will be flipped, making  $|eg\rangle \rightarrow |ge\rangle$  and  $|ge\rangle \rightarrow |eg\rangle$ .

Please note that  $\hat{U}(\theta)$  is not the general form of Walsh-Hadamard gate although it plays a similar role to the Walsh-Hadamard gate. So we introduce a two-qubit operation

$$\hat{M}_1^{(2)} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \end{pmatrix}, \text{ which plays similar role to the controlled-NOT gate. In what follows, we implement a two-qubit Grover search with above operations as an example.}$$

The most efficient Grover search includes three kinds of operations in an iteration(i.e., a searching step)<sup>[3]</sup>, (i) preparing a superposition of states with equal amplitude; (ii) inverting the amplitude of the marked state; (iii) performing a diffusion transform  $\hat{D}$ , i.e., the inversion about average(IAA) operation, with  $\hat{D}_{ij} = \frac{2}{N}$  for  $i \neq j$  and  $N = 2^q$ ( $q$  being the number of the qubits), and  $\hat{D}_{ii} = -1 + \frac{2}{N}$ . With our method, we first prepare two ion-pairs to the states  $|ge\rangle_1|ge\rangle_2$ , i.e.,  $|1\rangle_1|1\rangle_2$ (labeled as  $|11\rangle$  for simplicity in the following), without consideration of the vibrational states. Then  $\hat{U}(\frac{7\pi}{4})$  will be performed on the two pairs simultaneously, we obtain

$$|\Psi_1\rangle = \hat{W}_2 \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} -1 \\ i \\ i \\ 1 \end{pmatrix} \quad (2)$$

with

$$\hat{W}_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix} \otimes \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & i & i & -1 \\ i & 1 & -1 & i \\ i & -1 & 1 & i \\ -1 & i & i & 1 \end{pmatrix}.$$

As  $\hat{U}(\theta)$  is not the Walsh-Hadamard gate,  $|\Psi_1\rangle$  is not the superposition of states as the original Grover method required<sup>[3]</sup>. But that does not matter. We can still continue the procedure in the Grover search. Supposing that the marked state is  $|11\rangle$ , we have to invert the amplitude of this state, that is,

$$|\Psi_2\rangle = \hat{P}_1^{(2)} \frac{1}{2} \begin{pmatrix} -1 \\ i \\ i \\ 1 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} -1 \\ i \\ i \\ -1 \end{pmatrix} \quad (3)$$

where  $\hat{P}_1^{(2)} = \hat{V}_2^{-1} \hat{M}_1^{(2)} \hat{V}_2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$  with  $\hat{V}_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}$ . The operation  $\hat{V}_2$  means that  $\hat{U}(\frac{7\pi}{4})$  is performed on the second pair, whereas no operation on the

first pair. Finally, the IAA operation in the Grover search can be realized by the operation

$$\hat{D}_2 = \hat{W}_2 \hat{P}_1^{(2)} \hat{W}_2 = \frac{1}{2} \begin{pmatrix} -1 & i & i & -1 \\ i & 1 & -1 & -i \\ i & -1 & 1 & -i \\ -1 & -i & -i & -1 \end{pmatrix}. \quad (4)$$

It is easily found that

$$|\Psi_3\rangle = \hat{D}_2 |\Psi_2\rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \quad (5)$$

which means that the state  $|11\rangle$  has been found out. For further search, we may find that the state  $(0001)^{-1}$  recurs every third search steps, same as the demonstration in Ref.[4].

According to the Grover search<sup>[3]</sup>, one can find out a certain state by the operation of IAA as long as the amplitude of that state has been inverted. It can be found that our scheme also meets this requirement. Defining other three two-qubit operations to be

$$\hat{M}_2^{(2)} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & i \\ 0 & 0 & -i & 0 \end{pmatrix}, \quad \hat{M}_3^{(2)} = \begin{pmatrix} 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad \text{and} \quad \hat{M}_4^{(2)} = \begin{pmatrix} 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad \text{the}$$

inversion operations will be  $\hat{P}_2^{(2)} = \hat{V}_2^{-1} \hat{M}_2^{(2)} \hat{V}_2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ ,  $\hat{P}_3^{(2)} = \hat{V}_2^{-1} \hat{M}_3^{(2)} \hat{V}_2 = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ . If we want to find out a

$\hat{P}_i^{(2)}$  and  $\hat{P}_4^{(2)} = \hat{V}_2^{-1} \hat{M}_4^{(2)} \hat{V}_2 = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ . If we want to find out a

certain  $i$ th state, the search process will be the same as the above, except that a specific  $\hat{P}_i^{(2)}$  operation is made to invert the amplitude of the  $i$ th state. That is to say, no matter which state is to be searched, the IAA operation is still  $\hat{D}_2 = \hat{W}_2 \hat{P}_1^{(2)} \hat{W}_2$ . But we need a

specific operation  $\hat{P}_i^{(2)}$  to invert the amplitude of the  $i$ th state before each IAA operation.

### III. MORE QUBITS GROVER SEARCH

Along the idea in last section, we can extend the technique to the many-qubit cases for the Grover search. For a q-qubit case, we should first construct the Walsh-Hadamard gate as follows

$$\hat{W}_q = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix} \otimes \cdots \otimes \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix} \quad (6)$$

where  $\cdots$  represents the tensor product of  $q-2$  terms, and then the IAA operation is  $\hat{D}_q =$

$$\hat{W}_q \hat{P}_1^{(q)} \hat{W}_q \text{ with the } 2^q \times 2^q \text{ matrix } \hat{P}_1^{(q)} = \begin{pmatrix} 1 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & 1 & 0 \\ 0 & 0 & \dots & 0 & -1 \end{pmatrix}. \hat{P}_1^{(q)}$$

can be implemented

$$\text{by the operations } \hat{V}_q^{-1} \hat{M}_1^{(q)} \hat{V}_q \text{ with } 2^q \times 2^q \text{ matrix } \hat{M}_1^{(q)} = \begin{pmatrix} 1 & 0 & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & 1 & 0 & 0 \\ 0 & 0 & \dots & 0 & 0 & -i \\ 0 & 0 & \dots & 0 & i & 0 \end{pmatrix} \text{ and}$$

$$\hat{V}_q = \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes \cdots \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}_{q-1 \text{ terms}} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}.$$

To invert the amplitude of the marked state,

we need not only  $\hat{P}_1^{(q)}$ , but also  $\hat{P}_i^{(q)} (i = 2, 3, \dots, 2^q)$ . These matrixes can be constructed similarly. With the operations  $\hat{W}_q$ ,  $\hat{P}_i^{(q)}$  and  $\hat{D}_q$ , we can carry out the many-qubit Grover search.

However we have no way to present a general expression for the result of each iteration of a many-qubit Grover search with the present scheme as did in Ref.[17]. So we only investigated specifically the 2~5 qubits Grover search with the scheme by means of specific calculations with Mathematica. For the case of  $N = 3$ , the Walsh-Hadamard gate is  $\hat{W}_3 =$

$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix} \otimes \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix} \otimes \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}$  and the IAA operation is

$$D_3 = \frac{1}{8} \begin{pmatrix} 1 & -i & -i & -1 & -i & -1 & -1 & -3i \\ -i & -1 & -1 & i & -1 & i & -3i & 1 \\ -i & -1 & -1 & i & -1 & -3i & i & 1 \\ -1 & i & i & 1 & -3i & 1 & 1 & -i \\ -i & -1 & -1 & -3i & -1 & i & i & 1 \\ -1 & i & -3i & 1 & i & 1 & 1 & -i \\ -1 & -3i & i & 1 & i & 1 & 1 & -i \\ -3i & 1 & 1 & -i & 1 & -i & -i & -1 \end{pmatrix}.$$

We use  $|111\rangle$  as the initial state, and suppose the marked state is also  $|111\rangle$ . The result of each iteration of the search is plotted in Fig.1. We may find that, compare to the standard iteration process of the Grover search<sup>[3,17]</sup>, although we also have large probabilities (i.e.  $\geq 50\%$ ) to find out the desired state successfully within  $\sqrt{8}$  steps, the searching result with our scheme is no longer strictly periodic. The numerical result also showed that, the amplitude of the state  $|000\rangle$  always equals that of  $i|111\rangle$  in the searching process. It means that we will acquire two results simultaneously by direct measurement, and one of them has to be deleted as an undesired solution. It adds the searching steps and will much lower the efficiency of the Grover search if the multiply maximal probabilities will appear in the many-qubit Grover search. Fortunately, such problem was not found in investigating the 4-qubit and 5-qubit cases. From Figs.2 and 3, we also know that, like 3-qubit case, the searches succeed within  $\sqrt{N}$  ( $N = 2^4$  and  $2^5$ ) steps, and the results of the searches are also not strictly periodic. It may be speculated that the problem of multiply maximal probabilities will no longer take place for the more qubits Grover search. Therefore, with our scheme, the maximal number of iteration for a successful search is still  $\sqrt{N}$  except the 3-qubit case which needs additonal steps for judging the correct solution.

#### IV. DISCUSSION AND CONCLUSION

Before discussing the advantages and limitation of our scheme, we should mention that the search we described above is incomplete. For a full Grover search, we must first test which states will be the marked states in order to adjust the phases of them<sup>[17–20]</sup>, which is also called the process of search criterion calculation<sup>[19]</sup>. This calculation can be implemented by introducing the quantum random number generator(QRNG) and some extra qubits, as well as the corresponding software - quantum program<sup>[20]</sup>. The QRNG can generate a random number in the binary representation. By comparing the number with the binary function needed to be satisfied, the signs of the amplitudes of states would be determined without any external influence. It is obvious that the Walsh-Hadamard-type gate can be used as the QRNG, and the search criterion calculation can be carried out polynomially under the mechanism of quantum parallelism. Moreover, in a tighter analysis of the Grover search<sup>[17]</sup>, the iteration of the search was described strictly mathematically. It is shown that, as long as we know the total number of items in the searched database and the number of the solutions, the optimal searching steps can be calculated beforehand and the search can thereby be made efficiently<sup>[17,18]</sup>. However, the calculation for estimating the number of the solutions is not always necessary before the search is implemented. In practice, we can use the straightforward algorithm proposed in the Lemma 2 of Ref.[17] by introducing some random processes, which has been proven to be polynomial. So from above discussion, we can know that our scheme is still practical for the full Grover search. Actually, as only few-qubit quantum computing has been carried out experimentally so far, no considerations for the search criterion calculation and multi-solution problem have been taken in the experimental implementation of the search<sup>[4,5]</sup>. The quantum computing hardware is still in its infancy. We also note that the usefulness of the Grover search in the practical application is questioned<sup>[19]</sup>. As more specific discussion along this direction is beyond the scope of the paper, in what follows, our discussion will be restricted in analyzing the limitation and advantage of the scheme described in Sec.II and III.

The power of the Grover search increases with the increase of the number of qubits. So a practical scheme for the search should work in the case of large numbers of qubits. While with our method, we did not demonstrate a general expression for each iteration of the Grover search with arbitrary numbers of qubits. The specific numerical calculation was only made for the few-qubit cases. Moreover, we have not found how to implement  $\hat{M}_i^{(q)}$  ( $q \geq 3$ ) with two-qubit operations  $\hat{U}$  and  $\hat{M}_i^{(2)}$  in our scheme so far. Nevertheless, according to the general discussion in Ref.[14], we can definitely achieve the quantum computing when we have a single-qubit and a two-qubit unitary operations. Particularly, from the theorem 3 in Ref.[18], we know that, as long as we have a Walsh-Hadamard-type operation, the Grover's search can be definitely achieved. So we consider that our scheme can have the quadratic speedup not only for the few-qubit cases shown specifically in last section, but also for cases with arbitrarily large numbers of qubits. Furthermore, we noted that, the general form of the Walsh-Hadamard gate should have the form of SU(2). At least the gate should be with the form of 'y-axis rotation matrix'  $\begin{pmatrix} \cos(\theta/2) & \sin(\theta/2) \\ -\sin(\theta/2) & \cos(\theta/2) \end{pmatrix}$ . However,  $\hat{U}(\theta)$  in our paper is a 'x-axis rotation matrix'<sup>[14,21]</sup>, a special unitary operation, we consider that it is the reason results in the non-periodic iteration in our scheme.

More importantly, we can find following advantages of our scheme:

- (i) the undesired phase factors produced during the delay period between any two operations will turn to the global phase due to the degeneracy of the logic states, which makes the actual implementation of an ideal quantum computing available;
- (ii) our scheme based on Ref.[12] still meets the requirement of hot-ion quantum computing and scalability. As the vibrational states of the ions are decoupled from the internal states of the ions, the quantum information may be processed and transferred nearly safely in the subspace spanned by the internal states of the ion-pairs;
- (iii) even if we assume the decoherence will probably take place in the actual ion trap experiments due to some unpredictable factors such as the intensity fluctuations in the Raman laser beams etc<sup>[13]</sup>, the qubits with the ion pairs may be immune against any possible

decoherence caused by the surrounding environment. The two ions in a pair can be assumed to be decoherenced collectively because their distance is much smaller than the effective wave length of the thermal noise field<sup>[22]</sup>. As proposed in Ref.[23], by suitably choosing the intensity and the phase of a driving field, such a qubit can be in a coherent-preserving state which undergoes no decoherence even if it is interacted with the environment. It is worth being noticed that, the recent experiment<sup>[24]</sup> with polarization entangled states of photons has produced the coherent-preserving states. So we can expect that such a robust state will soon be produced in the ion trap experiment;

(vi) besides the decoherence effect, there is another detrimental effect, i.e., the dissipation<sup>[25]</sup> in the interaction between the trapped ions and the environment. In our scheme, the dissipation effect will be strongly suppressed due to the degeneracy of the logic states.

Now we make some discussions of the more technical aspects for the physical realization of our scheme in the ion trap. As reported in Ref.[13], four ultracold ions have been entangled in a linear ion trap by using the approach of bichromatic fields, and much larger numbers of ions can be entangled in principle with the same technique. With our scheme, we set the  $2N$  trapped ultracold ions to be  $N$  qubits, and choose  $|ge\rangle$  to be the initial state in each pair. For each ion, a very weak laser beam is needed to detect the quantum jumps in the internal states of the ion. Such a detection is within the reach of the present ion trap technique<sup>[26]</sup>, which presents us information about in which internal state the ion is and causes negligible influence on the original process. Although they are identical, the ions are distinguishable as long as the spacing between any two of them is in the order of magnitude of  $\mu m$ <sup>[11]</sup>, which is much larger than the size of the trap ground state( $10^{-9}m$ )<sup>[7]</sup>. To carry out operation  $\hat{U}$ , we only need to implement Eq.(1) with suitable choice of time and certain ion-pairs. However, to achieve operations  $\hat{M}_i^{(q)}$ , the situation would be somewhat complicated. We take  $\hat{M}_1^{(2)}$  as an example. If the first and second ions in the control pair (i.e., the ion pair acted as the control qubit) are in  $|g\rangle$  and  $|e\rangle$  respectively, the operation  $\hat{M}_1^{(2)}$  will be

the implementation of  $\hat{U}(\frac{3\pi}{2})(\hat{U}(\frac{\pi}{2}))$  on the target ion-pair when the target ion-pair is in  $|eg\rangle$  ( $|ge\rangle$ ).

In summary, an approach with pairs of trapped ions to achieve the Grover search has been proposed. As the logic states  $|0\rangle$  and  $|1\rangle$  are degenerated in energy, the relative phase caused by the free evolution of the states can be resorted to a global one, and thereby the detrimental impact on the quantum interference in the quantum algorithm can be completely diminished. We indicated that our scheme still meets the requirement for the full Grover search although the iteration of the search is not strictly periodic as the original Grover's approach. However, it is unclear in our scheme how to implement  $\hat{M}_i^{(q)}$  for  $q \geq 3$  more simply and efficiently. Moreover, the number of the ions required for quantum computing in our scheme is doubled compare to the former approaches, which is in some sense uneconomic, particularly for the fact that it is an uneasy task for cooling down a few ions in the existing ion trap experiments. Nevertheless, our scheme is applicable and useful due to the advantages listed above. With the fast development of the ion trap technique, we believe that much more entangled ions would be achieved in the linear ion trap in the near future. Therefore, our scheme is a promising one for the hot-ion quantum computing.

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### Captions of the figures

Fig.1 The probabilities of finding the marked state  $|111\rangle$  vs the number of iteration.

As the amplitude of the state  $|000\rangle$  is the same as that of  $|111\rangle$  in the iteration of the search, we have two readout results in this case. See text. The similar result can be obtained in searching for other states.

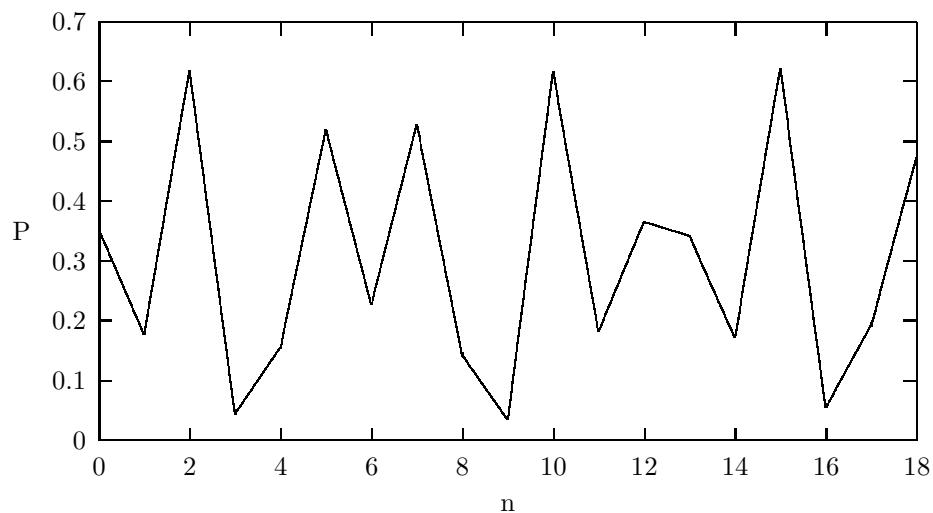
Fig.2 The probabilities of finding the marked state  $|1111\rangle$  vs the number of iteration.

The similar result can be obtained in searching for other states.

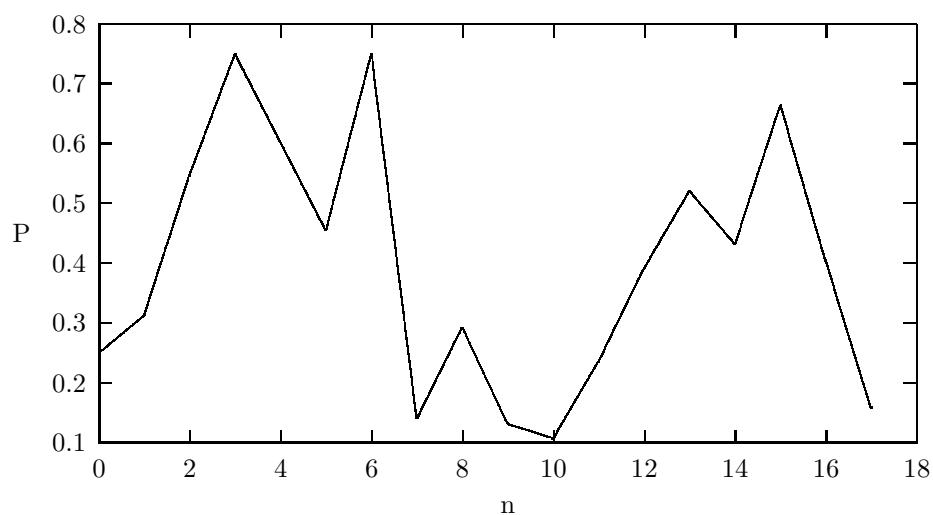
Fig.3 The probabilities of finding the marked state  $|11111\rangle$  vs the number of iteration.

The similar result can be obtained in searching for other states.

**Fig.1**



**Fig.2**



**Fig.3**

